Proceedings of the Japan International Tribology Conference
Nagoya, 1990

A KINEMATIC LAW OF FRICTION. THE PROOF AND POSSIBLE APPLICATIONS

Czesław KOZIARSKI*

A hypothesis put forward at the Eurotrib 81 Conference stating that slip velocity in the middle of an elementary contact area is reduced by the amount equal to velocity of elastic slip due to an elementary friction force turned out very useful in determining distributions of slip velocities and elementary friction forces within the contact area. By contrast to Coulomb law being in fact a dynamic law of friction the above statement constitutes a kinematic law of friction. It may be especially useful in simulation analysis of frictional coupling between two elastic bodies in low-speed sliding contact such as friction gear, wheel on rail or flat surface or tool guides. The kinematic law of friction has been generally accepted though no formal proof has ever been given. The proof presented here consists in demonstrating that the elastic slip velocity due to the elementary friction force is independent of the assumed size of the elementary contact area. It may be therefore implied that the law holds true for each individual point of contact area.

The paper also reports numerical results concerning the effect of elastic slip upon parameters of coupling between a tyre and base. There are also suggestions of other applications of the kinematic law.

1. INTRODUCTION

Slipping associated with external friction results from different velocities of two bodies at a point belonging to the surface of contact. It may be assumed that each of the two velocities is a sum of various components. The differences of the components may in turn be regarded as individual components of the slip velocity. Decomposition of a body velocities into components at some point of the contact surface can be performed in many ways depending on the reason why a particular component was originated or on other adopted criteria. Let us take an example of friction wheels. In this case a point of the contact zone not only moves along the circumference but is also displaced by the inherent difference of tensions. The difference arises due to the gradient of pressure occurring in that zone and generates the so called elastic slip. When other components of the contact zone point velocities are small enough to be neglected then elastic slip becomes responsible for direction and magnitude of elementary friction forces. The sum of these forces amounts to an effective force of friction coupling acting in a given direction. The problem seems to be especially important in friction gearing and tool guides. Friction disks not only roll upon each other but also slide at a speed which is much lower than the speed of rolling. In rolling the value of pressure in the contact zone varies and, as a consequence, so do values of elementary frictional forces. This gives rise to a varying tension mentioned already whose differential referred to a time increment defines velocity of elastic displacement. Such a displacement may be produced either by an elementary force of friction acting at a given point or by forces and pressures acting at other points of friction wheels. The differences in rolling velocities cause slipping called geometrical slipping. The differences in displacement velocities give rise to elastic slipping. The geometrical slip velocity is determined, as can be seen in Fig.1a, as a product of pivoting friction velocity ω_0 and position vector ρ , where

*Institute of Machine Design, Technical University of Wroclaw, 50-370 Wroclaw, Poland

$$\omega_0=\omega_1\sin\alpha_1-\omega_2\sin\alpha_2=\frac{\omega_1(R\mp R_2)}{(R_1\mp l)(R_2\mp l)} \qquad (1)$$

and ω_1, ω_2 - angular velocities of friction wheels
α_1, α_2 - edge angles of friction wheels
R_1, R_2 - rulings of cone of friction wheels

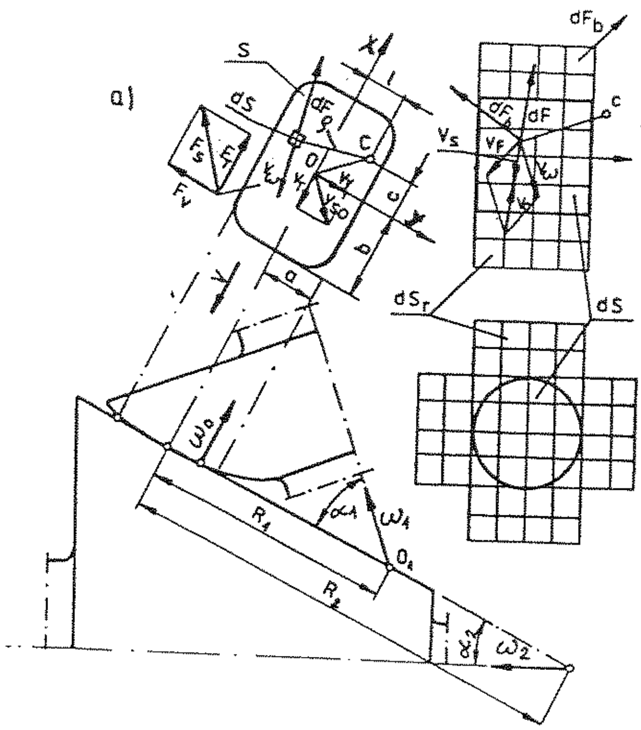

Fig.1 Model of mechanism of coupling between friction wheels
a) mechanism of coupling with the friction wheel surface considered absolutely rigid
b) model of coupling within a line contact zone
c) discrete model of a point contact zone

Position of rolling point C is given by its coordinates:
- distance along x axis according to [1]

$$l=v_T/\omega_0 \qquad (2)$$

where v_T - velocity of circumferential slip in the middle of the contact area of friction wheels
- deviation of the rolling point along y axis according to [2]

$$c=v_V/\omega_0 \qquad (3)$$

where v_V - velocity of slip between friction wheels along their line of contact.

Assuming a considered point to be the midpoint of an elementary contact area dS we prescribe to it an elementary friction force dF. A geometrical sum of these forces amounts to the force F_S of coupling between friction wheels. Neglecting elastic slip either for point contact (Lutz) or line contact (Wernitz) it is possible to determine circumferential forces which in combination with longitudinal forces F_V [2] give the force of friction coupling

$$F_S=\sqrt{F_T^2+F_V^2} \qquad (4)$$

Elementary friction forces also produce a moment of pivoting friction T_0 and some deformation of the contact surface. The latter in turn gives rise to elastic slip deviating friction forces directions in way shown in Fig.1b. The effect of elastic slip upon the mechanism of coupling between a rail and a driven wheel of a locomotive has been analyzed by Kalker [3,4] who used a method based on seeking a minimum of the sum of squares of potential functions to work out computer programs for determining forces and moments within the point contact area. The original Kalker method requires however a lot of laborious computational work [3]. In order to facilitate the computations Kalker proposed the linear theory assuming simplified relationships between forces, slip velocities along x and y axes and pivoting friction velocities and valid for very small values of pivoting friction velocity

$$F_T=-abGC_{11}v_T,\quad F_V=-abG(C_{22}v_V+C_{22}\sqrt{ab}\,\omega_0) \qquad (5)$$

where:
a,b - semiaxes of the elliptic contact area
G - shear modulus
and coefficients c_{ij} depend on Poisson's ratio value and shape of elliptic contact zone, F - total friction force.
In formulas (5) values of forces do not depend upon values of perpendicular slip and force F_T is independent of ω_0. For contact between a rail and wheel the assumption is justified by the observation that V_V and ω_0 are negligibly small when compared with circumferential velocity v. Analysis of the published data demonstrates another advantage of the linear Kalker theory consisting in that it is the only method enabling determination of forces in pivoting friction when $(\omega_0/v)\to 0$. Operation of variable-speed friction gears usually involves much higher velocities V_V and ω_0 than those found in the rail-wheel case. That is why the linear theory is of little interest there. Furthermore, Kalker's programs for more accurate analysis are not

available in the open literature [3]. It should also be added that experimental evidence supporting the linear theory is still scarce (mainly Johnson [5]) and attempts are being made to extend the theory´s scope.

A modified approach is used to examine the effect the elastic slipping exerts on coupling of friction wheels in which an an elementary friction force dF is assumed to act at the midpoint of an elementary contact area dS. The vectors of force and slip velocity attached to the area midpoint act in reverse directions. Initially the velocity is a sum of all velocity components except a slip velocity due to force dF. An example is shown in Fig.1b where v_ω is the geometric slip velocity and v_F - the elastic slip velocity at this point caused by a change in displacements with the elementary force dF being disregarded. A sum of those slips, which may as well be constituted by differences of other velocities, determines direction of the slip velocity. If the elastic slip velocity v_0 due to force dF does not exceed the sum of other slips, then subtracting it from that sum gives the actual slip velocity v_S. Otherwise it is assumed that $v_S=0$ and an elementary friction force dF is modified by lovering its value in proportion to the slip velocity value v_0. It means that at a point of contact between two bodies slip occurs only if the elementary slip velocity due to the elementary friction force is smaller than the sum of slip velocities due to other causes.

The above assumption was first formulated in [2]. By contrast to Coulomb´s law involving forces arising in sliding between two bodies and being in fact a dynamic law of friction the above statement involves velocities and as such may be regarded a kinematic law of friction.

2. A KINEMATIC LAW OF FRICTION - THE PROOF

Let us express an elastic slip velocity due to an elementary force of friction dF as

$$v_0 = \frac{du_1}{dt} + \frac{du_2}{dt} = \frac{u_1+u_2}{\Delta t} \qquad (6)$$

where u_1, u_2 - displacements of point of contact of friction wheels due to force dF on the plane of contact

and a displacement time as

$$t = \frac{2\Delta}{v} \qquad (7)$$

where Δ - half of elementary contact area length

v - displacement velocity

Assuming for simplicity that one friction disc is rigid on its surface and that elementary contact area is square in form and the elementary force F acts in such a way as to produce uniform tangential loading $p\mu$ directed along x axis, we can write Cerruti formula as

$$U = \frac{p\mu\,(1+\nu)}{\pi E} \int_{dS} \frac{x^2+y^2-\nu y^2}{\sqrt{(x^2+y^2)^3}}\,dxdy \qquad (8)$$

where

ν - Poisson´s ratio

μ - friction coefficient

E - elasticity modulus

p - pressure

Inserting relationships (7) and (8) into (6) we get

$$v_0 = \frac{p\mu v(1+\nu)}{2\pi E\Delta} \int_{-\Delta}^{\Delta}\int_{-\Delta}^{\Delta} \frac{x^2+y^2-\nu y^2}{\sqrt{(x^2+y^2)^3}}\,dxdy \qquad (9)$$

Multiplying geometrical quantities in the above equation by a scaling coefficient β we arrive at an unchanged form of the equation. It serves an evidence that the elastic slip velocity due to the elementary friction force is independent of size of the elementary contact area. It can be therefore implied that the kinematic law of friction is valid for each selected point of contact.

Assumption of a specific size of contact area has direct effect upon accuracy of calculations, computer time consumption and possibility of convergence enhancement in case iterative calculations are adopted. The iterative method is justified by nonlinear relationships between forces and displacements on the contact plane. Application of the kinematic law of friction in iterative calculating of the parameters of friction gear coupling leads to the Newton method of convergence enhancement. This so called super-relaxation method is less effective than the method of seeking a minimum sum of squares of potential functions but the calculations are faster.

3. COMPUTATIONAL RESULTS, CONCLUSIONS AND APPLICATIONS

By contrast to rail traction the friction gear is characterized by a linear contact zone between the wheels. One wheel of the pair is covered with frictional material whose elasticity modulus is from several hundred to several thousand times smaller than Young´s modulus of steel. It is therefore clear that deformation of the second baze steel wheel can be safely neglected. Moreover, for rubber bands in which elastic slip is a significant factor the Poisson´s ratio is nearly equal to 0.5. By taking this value we arrive at a simplified form of Cerruti formula and in consequence the amount of

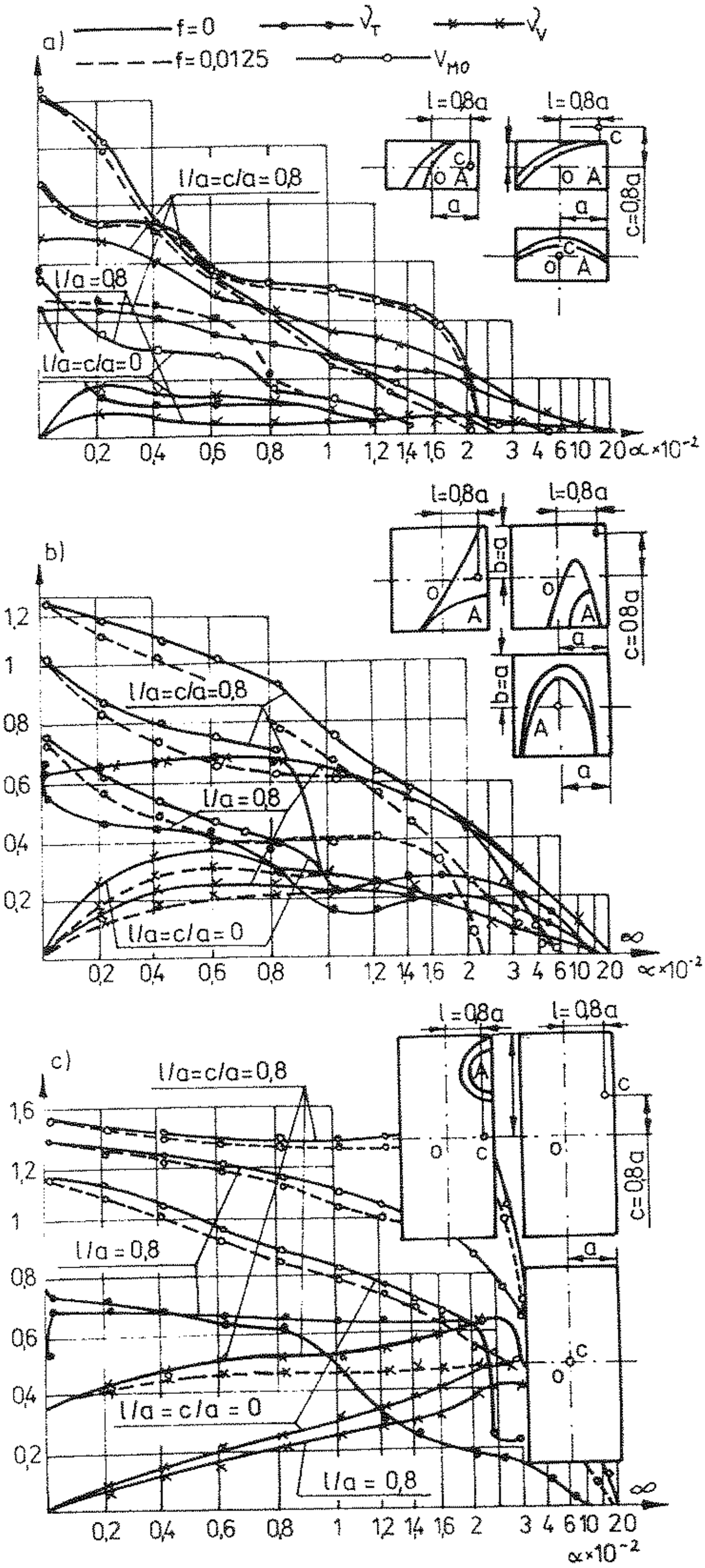

Fig.2 Calculated values of main coupling parameters for a tyre in contact with base
a) for b/a=0,5
b) for a=1
c) for b/a=2

computational work becomes smaller. In the present study differences of speeds at which deformation of the frictional band occurred were taken equal to accelerations of elementary contact areas. Influence of elastic slip on the mechanism of frictional coupling between two wheels in linear rolling contact was determined and shown in Fig.2. The calculations were performed by prescribing elementary masses to elementary contact areas dS and elementary areas dS_r adjacent to the first ones (see Fig.1b and 1c). A limiting assumption of planar inertia forces is justified by the fact that the friction band thickness is small so that longitudinal and transverse waves can hardly propagate. Damping of displacements deep inside the friction band material is much stronger than that in an elastic semispace due to limited size of the band and properties of the material. Dimensional limitations imposed on the semispace also result in that the actual band deformation values are smaller than those suggested by formulas of Hertz and Cerruti. It may be concluded that amplitudes of free vibrations of the friction band surface also turn out smaller. Assuming the effect of these vibrations upon parameters of coupling in a friction gear to become negligible it is possible to examine merely the effect of surface inertia forces connected with forced vibrations on the coupling parameters values.

It is to be noted that the presented model of a friction band bound with a rigid base can be easily extended to the case of rubber tread laid on unstretchable cord as is observed in a radial tyre rolling against hard surface. A different treatment is required however when it comes to distribution of pressure between that kind of tread and base. In modern tyres the pressure tends to be uniformly distributed over the whole area of contact [8].

The graphs in Figs 2a, 2b and 2c show the main parameters of coupling between a tyre and base as functions of a dimensionless coefficient $\alpha = \pi p \nu \mu (aE\omega_0)$, where p - pressure in contact zone, a- half of contact zone length, b - half of contact zone width, E - tread elasticity modulus.
Notation used in Fig.2a reads as follows:
- dimensionless mass index

$$f = \frac{dF_b}{\frac{\partial V}{\partial t}\, dS} ,$$

where dF_b - elementary surface inertia force
fraction of effective (usable) friction force in circumferential direction $\nu_T = F_T/F$ and in line of contact direction

$$\nu_v = \frac{F_v}{F}$$

- dimensionless power loss coefficient

$$\nu_{M_o} = \frac{T_o}{aF}$$

The calculations presented in Fig.2 were accomplished with the tangent force distribution within the friction gear contact zone taken as uniform. It is plain from the figures that elastic slip has a strong effect on main parameters of coupling between a tyre and base and between friction wheels operating in line and point contact [1,7]. Surface inertia forces markedly increase the effect of elastic slip on ν_{M_o} value and only slightly on the fraction of usable friction force ν_T and ν_V. At top right-hand corners of Figs 2 depicted are shapes of contact zones with superimposed areas of adhesion (A) into which rolling points C were converted owing to the action of elastic slip. Areas A were determined for α= 101. Their size increases with increasing α and for f=0.0125 they are larger than those determined for f=0.

Fig.3 presents distribution of slip velocities v and elementary friction forces dF, dF_b within the tyre-base contact area. It is clear from the figure that elastic slip has some effect on mechanism of wheel coupling as does slip V_V along the line of contact [2]. Apart from decreasing the values of slips it displaced their centre of revolution perpendicularly to the wheel contact line thus giving rise to longitudinal force F_V similarly as did deviation C of a rolling point. By contrast to the friction wheel coupling where inertia forces dF_b have a disturbing effect upon directions and magnitudes of slip velocities as acting in the direction that is usually opposite to the slip direction (near the centre) and coinciding in direction farther from the centre in the tyre they act practically in front of the contact area or behind it. This is due to small displacements within this area. Greater role of surface inertia forces may be expected in analysis of vibrations arising in tool guides.

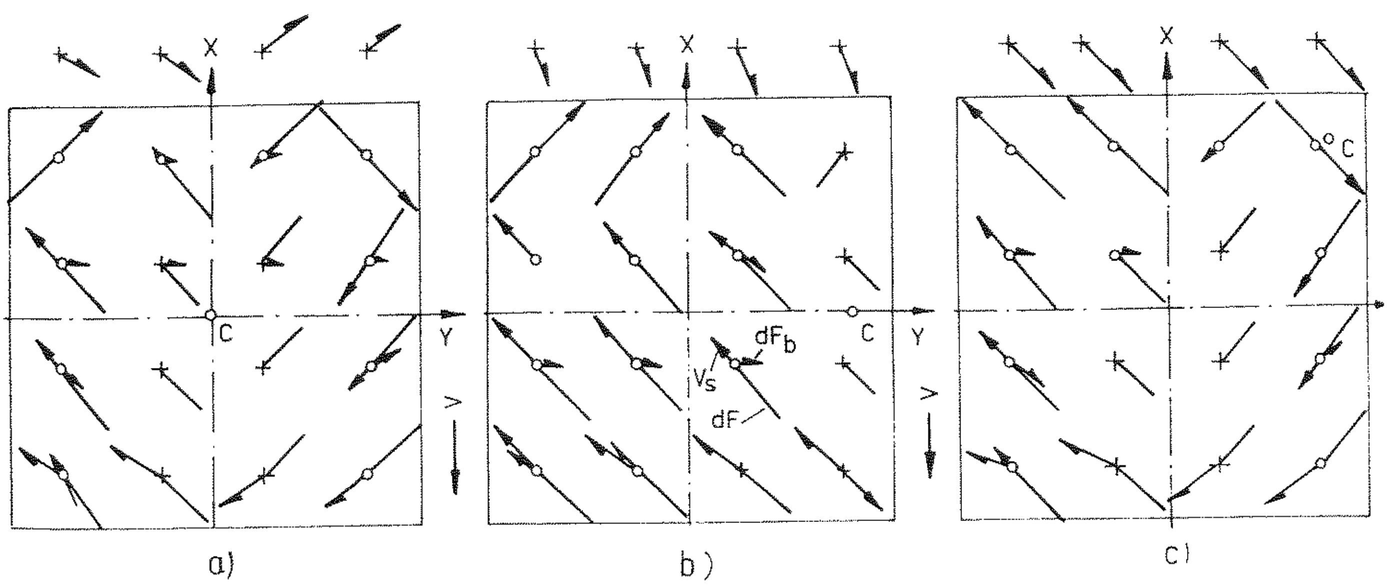

Fig.3 Distribution of slip velocities, elementary friction forces and inertia forces within the tyre-base contact zone for b/a=1

REFERENCES

[1] Cz. Koziarski : 13 th Conference on "Models in Designing an Constructions of Machines". Politechnika Śląska Z.92, Gliwice (1989) 141.

[2] Cz. Koziarski : 4 th European Tribology - Congress EUROTRIB 85, Ecully - France, Elsevier (1985) Vol. III, 5.1.

[3] J. J. Kalker : Vehicle System Dynamics 5 (1979) 317.

[4] J. J. Kalker : Conference on Theoretical and experimental foundations of life prediction of pair wheel-rail, Spała - Poland (1987) Vol. 1.2.

[5] K. L. Johnson and J. B. Bidwell : Proc. Symp. Rolling Contact Phenomena, Elsevier (1962) 6.

[6] Cz. Koziarski : 3 rd International Tribology Congress EUROTRIB 81, Warszawa, WSI Radom (1981) Vol. 1/A 227.

[7] Cz. Koziarski : 5 th International Congress on Tribology EUROTRIB 89, Espoo Helsinki (1989) 151.

[8] H. Sakai : Int. J. of Vehicle Design, Vol. 2 no. 2 (1981) 182.